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11.

$$\int\sqrt{\frac{a+x}{a-x}}$$

Solution:

$$\text{Let,}\quad I = \int \sqrt{\frac{a+x}{a-x}}\, dx$$

$$= \int \sqrt{\frac{a+x}{a-x} \times \frac{a+x}{a+x}}\, dx = \int \frac{a+x}{\sqrt{(a-x)(a+x)}}\, dx$$

$$= \int \frac{a+x}{\sqrt{a^2-x^2}}\, dx = \int \frac{a}{\sqrt{a^2-x^2}}\, dx + \int \frac{x}{\sqrt{a^2-x^2}}\, dx$$

Let's consider, $I = I_1 + I_2$

$$\text{Now,}\ I_1 = \int \frac{a}{\sqrt{a^2-x^2}}\, dx = a.\sin^{-1}\frac{x}{a} + C_1$$

$$\text{And,}\ I_2 = \int \frac{x}{\sqrt{a^2-x^2}}\, dx$$

$$\text{Putting,}\quad a^2 - x^2 = t \Rightarrow -2x\, dx = dt$$

$$x\, dx = \frac{dt}{-2}$$

$$\text{Hence,}\ I_2 = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \times 2\sqrt{t} = -\sqrt{a^2-x^2} + C_2$$

$$\text{As,}\ I = I_1 + I_2$$

$$= a\sin^{-1}\frac{x}{a} + C_1 - \sqrt{a^2-x^2} + C_2$$

$$\therefore\ I = a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + (C_1 + C_2)$$

$$\text{Therefore,}\ I = a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + C \qquad [C = C_1 + C_2]$$

Let, $\quad I = \int \sqrt{\frac{a+x}{a-x}}\, dx$

$$= \int \sqrt{\frac{a+x}{a-x} \times \frac{a+x}{a+x}}\, dx = \int \frac{a+x}{\sqrt{(a-x)(a+x)}}\, dx$$

$$= \int \frac{a+x}{\sqrt{a^2-x^2}}\, dx = \int \frac{a}{\sqrt{a^2-x^2}}\, dx + \int \frac{x}{\sqrt{a^2-x^2}}\, dx$$

Let's consider, $I = I_1 + I_2$

Now, $I_1 = \int \frac{a}{\sqrt{a^2-x^2}}\, dx = a.\sin^{-1}\frac{x}{a} + C_1$

And, $I_2 = \int \frac{x}{\sqrt{a^2-x^2}}\, dx$

Putting, $a^2 - x^2 = t \Rightarrow -2x\, dx = dt$

$$x\, dx = \frac{dt}{-2}$$

Hence, $I_2 = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \times 2\sqrt{t} = -\sqrt{a^2-x^2} + C_2$

As, $I = I_1 + I_2$

$$= a\sin^{-1}\frac{x}{a} + C_1 - \sqrt{a^2-x^2} + C_2$$

$$\therefore\ I = a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + (C_1 + C_2)$$

Therefore, $I = a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + C \qquad [C = C_1 + C_2]$

12.

$$\int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}}\,dx \qquad (\textbf{Hint}: \text{Put } x = z^4)$$

Solution:

Let, $I = \int \frac{x^{1/2}}{1+x^{3/4}}\,dx$

$$\begin{array}{r} t^3+1\,\big)\, t^5 \,\big(\, t^2 \\ t^5 + t^2 \\ (-)\ \ (-) \\ \hline -t^2 \end{array}$$

Putting, $x = t^4 \Rightarrow dx = 4t^3\,dt$

$$= \int \frac{t^2 . 4t^3}{1+t^3}\,dt = 4\int \frac{t^5}{1+t^3}\,dt$$

$$= 4\int \left(t^2 - \frac{t^2}{t^3+1}\right) dt = 4\int t^2\,dt - 4\int \frac{t^2}{t^3+1}.\,dt$$

Let's consider, $I = I_1 - I_2$

Now, $I_1 = 4\int t^2\,dt = 4.\frac{t^3}{3} + C_1 = \frac{4}{3}x^{3/4} + C_1$

$$I_2 = 4\int \frac{t^2}{t^3+1}\,dt$$

Putting, $t^3 + 1 = z \Rightarrow 3t^2\,dt = dz$

$$t^2\,dt = \frac{1}{3}dz$$

So, $I_2 = \frac{4}{3}\int \frac{dz}{z} = \frac{4}{3}\log|z| + C_2 = \frac{4}{3}\log\left|t^3+1\right| + C_2$

$$= \frac{4}{3}\log\left|(x)^{3/4}+1\right| + C_2$$

As, $I = I_1 - I_2$

$$= \frac{4}{3}x^{3/4} + C_1 - \frac{4}{3}\log\left|(x)^{3/4}+1\right| - C_2$$

$$= \frac{4}{3}\left[x^{3/4} - \log\left|(x)^{3/4}+1\right|\right] + C_1 - C_2$$

Therefore, $I = \frac{4}{3}\left[x^{3/4} - \log\left|(x)^{3/4}+1\right|\right] + C \qquad [\because C = C_1 - C_2$

13.

$$\int \frac{\sqrt{1+x^2}}{x^4}\,dx$$

Solution:

$$\text{Let, } I = \int \frac{\sqrt{1+x^2}}{x^4}\,dx$$

$$= \int \sqrt{\frac{1+x^2}{x^2}} \cdot \frac{1}{x^3}\,dx = \int \sqrt{\frac{1}{x^2}+1} \cdot \frac{1}{x^3}\,dx$$

$$\text{Taking, } \frac{1}{x^2} + 1 = t^2$$

$$\text{So, } \frac{-2}{x^3}\,dx = 2t\,dt \Rightarrow \frac{dx}{x^3} = -t\,dt$$

$$\therefore\ I = \int t(\quad t\,dt) = -\int t^2\,dt = -\frac{1}{3}t^3 + C$$

$$\text{Therefore, } I = -\frac{1}{3}\left(\frac{1}{x^2}+1\right)^{3/2} + C$$

14.

$$\int \frac{dx}{\sqrt{16-9x^2}}$$

Solution:

Let, $I = \int \frac{dx}{\sqrt{16-9x^2}}$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9}-x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{4}{3}\right)^2 - x^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{4/3} + C \quad \left[\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C\right]$$

$$= \frac{1}{3}\sin^{-1}\frac{3x}{4} + C$$

Therefore, $I = \frac{1}{3}\sin^{-1}\frac{3x}{4} + C.$

15.

$$\int \frac{dt}{\sqrt{3t-2t^2}}$$

Solution:

Let, $I = \int \frac{dt}{\sqrt{3t - 2t^2}} = \int \frac{dt}{\sqrt{-2\left(t^2 - \frac{3}{2}t\right)}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left(t^2 - \frac{3}{2}t + \frac{9}{16} - \frac{9}{16}\right)}}$$ [Making perfect square]

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left[\left(t - \frac{3}{4}\right)^2 - \frac{9}{16}\right]}} = \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\frac{9}{16} - \left(t - \frac{3}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2 - \left(t - \frac{3}{4}\right)^2}} = \frac{1}{\sqrt{2}}.\sin^{-1} \frac{t - \frac{3}{4}}{\frac{3}{4}} + C$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{4t - 3}{3} + C$$

Therefore, $I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4t - 3}{3}\right) + C.$

Let, $I = \int \frac{dt}{\sqrt{3t - 2t^2}} = \int \frac{dt}{\sqrt{-2\left(t^2 - \frac{3}{2}t\right)}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left(t^2 - \frac{3}{2}t + \frac{9}{16} - \frac{9}{16}\right)}}$$ [Making perfect square]

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left[\left(t - \frac{3}{4}\right)^2 - \frac{9}{16}\right]}} = \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\frac{9}{16} - \left(t - \frac{3}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2 - \left(t - \frac{3}{4}\right)^2}} = \frac{1}{\sqrt{2}} . \sin^{-1} \frac{t - \frac{3}{4}}{\frac{3}{4}} + C$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{4t - 3}{3} + C$$

Therefore, $I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4t - 3}{3}\right) + C.$